

Right-Handed Quark Mixings in Minimal Left-Right Symmetric Model with General CP Violation

Yue Zhang,^{1,2} Haipeng An,² Xiangdong Ji,^{2,1} and R. N. Mohapatra²

¹*Center for High-Energy Physics and Institute of Theoretical Physics,
Peking University, Beijing 100871, China*

²*Department of Physics, University of Maryland,
College Park, Maryland 20742, USA*

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Abstract

We present a systematic approach to solve analytically for the right-handed quark mixings in the minimal left-right symmetric model which generally has both explicit and spontaneous CP violations. The leading-order result has the same hierarchical structure as the left-handed CKM mixing, but with additional CP phases originating from a spontaneous CP-violating phase in the Higgs vev. We explore the phenomenology entailed by the new right-handed mixing matrix, particularly the bounds on the mass of W_R and the CP phase of the Higgs vev.

The physics beyond the standard model (SM) has been the central focus of high-energy phenomenology for more than three decades. Many proposals, including supersymmetry, technicolors, little Higgs, and extra dimensions, have been made and studied thoroughly in the literature; tests are soon to be made at the Large Hadron Collider (LHC). One of the earliest proposals, the left-right symmetric (LR) model [1], was motivated by the hypothesis that parity is a perfect symmetry at high-energy, and is broken spontaneously at low-energy due to an asymmetric vacuum. Asymptotic restoration of parity has a definite aesthetic appeal [2]. This model, based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, has a number of additional attractive features, including a natural explanation of the weak hypercharge in terms of baryon and lepton numbers [3], the existence of right-handed neutrinos, and the possibility of spontaneous CP (charge-conjugation-parity) violation (SCPV) [4]. The model can easily be constrained by low-energy physics and predict clear signatures at colliders [5]. It so far remains a decent possibility for new physics.

The LR modes are best constrained at low-energies by flavor-changing mixings and decays, particularly the CP violating observables. In making theoretical predictions, the major uncertainty comes from the unknown right-handed quark mixing matrix, conceptually similar to the left-handed quark Cabibbo-Kobayashi-Maskawa (CKM) mixing. The new mixing generally depends on 9 real parameters: 6 CP violation phases and 3 rotational angles. Over the years, two limiting cases of the model have usually been studied. The first case, “manifest left-right symmetry”, assumes that there is no SCPV, i.e., all Higgs vacuum expectation values (vev) are real. The quark mass matrices are then hermitian, and the left and right-handed quark mixings become identical, modulo the sign uncertainty from possible negative quark masses. The reality of the Higgs vev, however, does not survive radiative corrections which generate infinite renormalization. The second case, “pseudo-manifest left-right symmetry”, assumes that the CP violation comes entirely from spontaneous symmetry breaking (SSB) and that all Yukawa couplings are real [6]. Here the quark mass matrices are complex but symmetric, the right-handed quark mixing is related to the complex conjugate of the CKM matrix multiplied by additional CP phases. There are few studies of the model with general CP violation in the literature [7], with the exception of an extensive numerical study in Ref. [8] where solutions were generated through a Monte Carlo method.

In this paper, we report a systematic approach to solve analytically for the right-handed quark mixings in the minimal LR model with general CP violation. As is well-known, the model has a Higgs bi-doublet whose vev’s are complex, leading to both explicit and spontaneous CP violations. The approach is based on the fact that $m_t \gg m_b$ and hence the ratio of the two vev’s of the Higgs bi-doublet, $\xi = \kappa'/\kappa$, is small. In the leading-order in ξ , we find a linear matrix equation for the right-handed quark mixing which can readily be solved. We present an analytical solution of this equation valid to $\mathcal{O}(\lambda^3)$, where $\lambda = \sin \theta_C$ is the Cabibbo mixing parameter. The leading-order solution is very close to the left-handed CKM matrix, apart from additional phases that are fixed by ξ , the spontaneous CP phase α , and the quark masses. This explicit right-handed quark mixing allows definitive studies of the neutral meson mixing and CP-violating observables. We use the experimental data on kaon and B -meson mixings and neutron electrical dipole moment (EMD) to constrain the mass of W_R and the SCPV phase α .

The matter content of the LR model is the same as the standard model (SM), except for a right-handed neutrino for each family which, together with the right-handed charged lepton, forms a $SU(2)_R$ doublet. The Higgs sector contains a bi-doublet ϕ , which transforms like $(2, 2, 0)$ of the gauge group, and the left and right triplets $\Delta_{L,R}$, which transform as $(3, 1, 2)$

and $(1, 3, 2)$, respectively. The gauge group is broken spontaneously into the SM group $SU(2)_L \times U(1)_Y$ at scale v_R through the vev of Δ_R . The breaking of the SM group is accomplished through vev's of ϕ .

The most general renormalizable Higgs potential can be found in Ref. [9]. Only one of the parameters, α_2 , which describes an interaction between the bi-doublet and triplet Higgs, is complex, and induces an explicit CP violation in the Higgs potential. It is known in the literature that when this parameter is real, SCPV does not occur if the SM group is to be recovered in the decoupling limit $v_R \rightarrow \infty$ [9]. Without SCPV, the Yukawa couplings in the quark sector are hermitian, and we have the manifest left-right symmetry limit. Here we are interested in the general case when α_2 is complex. A complex α_2 allows spontaneous CP violation as well, generating a finite phase α for the vevs of ϕ ,

$$\langle \phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha} \end{pmatrix}. \quad (1)$$

In reference [10], a relation was derived between α and the phase δ_2 of α_2 ,

$$\alpha \sim \sin^{-1} \left(\frac{2|\alpha_2| \sin \delta_2}{\alpha_3} \xi \right), \quad (2)$$

where α_3 is another interaction parameter between the Higgs bi-doublet and triplets.

The quark masses in the model are generated from the Yukawa coupling,

$$\mathcal{L}_Y = \bar{q}(h\phi + \tilde{h}\tilde{\phi})q + \text{h.c.} \quad (3)$$

Parity symmetry ($\phi \rightarrow \phi^\dagger$, $q_L \rightarrow q_R$) constrains h and \tilde{h} be hermitian matrices. After SSB, the above lagrangian yields the following quark mass matrices,

$$\begin{aligned} M_u &= \kappa h + \kappa' e^{-i\alpha} \tilde{h} \\ M_d &= \kappa' e^{i\alpha} h + \kappa \tilde{h}. \end{aligned} \quad (4)$$

Because of the non-zero α , both M_u and M_d are non-hermitian. And therefore, the right-handed quark mixing can in principle very different from that of the left-hand counter part.

Since the top quark mass is much larger than that of down quark, one may assume, without loss of generality, $\kappa' \ll \kappa$, while at the same time \tilde{h} is at most the same order as h . We parameterize $\kappa'/\kappa = rm_b/m_t$, where r is a parameter of order unity. As a consequence, M_u is nearly hermitian, and one may neglect the second term to leading order in ξ . One can account for it systematically in ξ expansion if the precision of a calculation demands. Now h can be diagonalized by a unitary matrix U_u ,

$$M_u = U_u \hat{M}_u S U_u^\dagger = \kappa h, \quad (5)$$

where \hat{M}_u is $\text{diag}(m_u, m_c, m_t)$, and S is a diagonal sign matrix, $\text{diag}(s_u, s_c, s_t)$, satisfying $S^2 = 1$. Replacing the h -matrix in M_d by the above expression, one finds

$$e^{i\alpha} \xi \hat{M}_u + \kappa U_u \tilde{h} U_u^\dagger S = V_L \hat{M}_d V_R^\dagger \quad (6)$$

where \hat{M}_d is $\text{diag}(m_d, m_s, m_b)$, V_L is the CKM matrix and V_R is the right-handed mixing matrix that we are after. Two comments are in order. First, through redefinitions of quark fields, one can bring V_L to the standard CKM form with four parameters (3 rotations and

1 CP violating phase) and the above equation remains the same. Second, all parameters in the unitary matrix V_R are now physical, including 3 rotations and 6 CP-violating phases.

To make further progress, one uses the hermiticity condition for $U_u \tilde{h} U_u^\dagger$, which yields the following equation,

$$\hat{M}_d \hat{V}_R^\dagger - \hat{V}_R \hat{M}_d = 2i\xi \sin \alpha V_L^\dagger \hat{M}_u S V_L \quad (7)$$

where \hat{V}_R is the quotient between the left and right mixing $V_R = S V_L \hat{V}_R$. There are a total of 9 equations above, which are sufficient to solve 9 parameters in \hat{V}_R . It is interesting to note that if there is no SCPV, $\alpha = 0$, the solution is simply $V_R = S V_L \tilde{S}$, where \tilde{S} is another diagonal sign matrix, $\text{diag}(s_d, s_s, s_b)$, satisfying $\tilde{S}^2 = 1$. We recover the manifest left-right symmetry case.

The above linear equation can be solved using various methods. The simplest is to utilize the hierarchy between down-type-quark masses. Multiplying out the left-hand side and assuming \hat{V}_{Rij} and \hat{V}_{Rij}^* are of the same order, which can be justified posteriori, the solution is (for $\rho \sin \alpha \leq 1$)

$$\begin{aligned} \text{Im} \hat{V}_{R11} &= -r \sin \alpha \frac{m_b m_c}{m_d m_t} \lambda^2 \\ &\times \left(s_c + s_t \frac{m_t}{m_c} A^2 \lambda^4 ((1 - \rho)^2 + \eta^2) \right) \end{aligned} \quad (8)$$

$$\text{Im} \hat{V}_{R22} = -r \sin \alpha \frac{m_b m_c}{m_s m_t} \left(s_c + s_t \frac{m_t}{m_c} A^2 \lambda^4 \right) \quad (9)$$

$$\text{Im} \hat{V}_{R33} = -r \sin \alpha s_t \quad (10)$$

$$\begin{aligned} \hat{V}_{R12} &= 2ir \sin \alpha \frac{m_b m_c}{m_s m_t} \lambda \\ &\times \left(s_c + s_t \frac{m_t}{m_c} \lambda^4 A^2 (1 - \rho + i\eta) \right) \end{aligned} \quad (11)$$

$$\hat{V}_{R13} = -2ir \sin \alpha A \lambda^3 (1 - \rho + i\eta) s_t \quad (12)$$

$$\hat{V}_{R23} = 2ir \sin \alpha A \lambda^2 s_t, \quad (13)$$

where $\text{Im} \hat{V}_{R11}$, $\text{Im} \hat{V}_{R22}$, $\text{Im} \hat{V}_{R33}$, \hat{V}_{R12} , \hat{V}_{R23} , \hat{V}_{R13} are on the orders of λ , λ , 1, λ^2 , λ^2 , and λ^3 , respectively. The above solution allows us to construct entirely the right-handed mixing to order $\mathcal{O}(\lambda^3)$

$$V_R = P_U V P_D, \quad (14)$$

where the factors $P_U = \text{diag}(s_u, s_d \exp(2i\theta_2), s_t \exp(2i\theta_3))$, $P_D = \text{diag}(s_d \exp(i\theta_1), s_s \exp(-i\theta_2), s_b \exp(-i\theta_3))$, and

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 e^{-i2\theta_2} \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 e^{2i\theta_2} & 1 \end{pmatrix}; \quad (15)$$

with $\theta_i = \tilde{s}_i \sin^{-1} \text{Im} \hat{V}_{Rii}$. The pseudo-manifest limit is recovered when $\eta = 0$.

A few remarks about the above result are in order. First, the hierarchical structure of the mixing is similar to that of the CKM, namely 1-2 mixing is of order λ , 1-3 order λ^3 and 2-3 order λ^2 . Second, every element has a significant CP phase. The elements involving the first two families have CP phases of order λ , and the phases involving the third family are

of order 1. These phases are all related to the single SCPV phase α , and can produce rich phenomenology for K and B meson systems as well as the neutron EDM. Third, depending on signs of the quark masses, there are $2^5 = 32$ discrete solutions. Finally, using the right-handed mixing at leading order in ξ , one can construct \tilde{h} from Eq. (6) and solve M_u with a better approximation. The iteration yields a systematic expansion in ξ .

In the remainder of this paper, we consider the kaon and B -meson mixing as well as the neutron EDM. We will first study the contribution to the $K_L - K_S$ mass difference ΔM_K and derive an improved bound on the mass of right-handed gauge boson W_R , using the updated hadronic matrix elements and strange quark mass. Then we calculate the CP violation parameter ϵ in K_L decay and the neutron EDM, deriving an independent bound on M_{W_R} . Finally, we consider the implications of the model in the B -meson system, deriving yet another bound on M_{W_R} .

The leading non-SM contribution to the $K_0 - \bar{K}_0$ mixing comes from the $W_L - W_R$ box diagram and the tree-level flavor-changing, neutral-Higgs (FCNH) diagram[11, 12]. The latter contribution has the same sign as the former, and inversely proportional to the square of the FCNH boson masses. We assume large Higgs boson masses ($> 20\text{TeV}$) from a large α_3 in the Higgs potential so that the contribution to the mixing is negligible. Henceforth we concentrate on the box diagram only.

Because of the strong hierarchical structure in the left and right quark mixing, the $W_L - W_R$ box contribution to the kaon mixing comes mostly from the intermediate charm quark,

$$H_{12} = \frac{G_F}{2} \frac{\alpha_{\text{em}}}{4\pi \sin^2 \theta_W} 2\eta \lambda_c^{LR} \lambda_c^{RL} m_c^2 \quad (16)$$

$$\times [4(1 + \ln x_c) + \ln \eta] [(\bar{d}s)^2 - (\bar{d}\gamma_5 s)^2] + \text{h.c.}$$

where $x_c = m_c^2/M_{W_L}^2$, $\eta = M_{W_L}^2/M_{W_R}^2$, $\lambda_c^{RL} = V_{Rcd}^* V_{Lcs}$, and $\lambda_c^{LR} = V_{Lcd}^* V_{Rcs}$. The above result is very similar to that from the manifest-symmetry limit because the phases in V_{Rcd} and V_{Rcs} are $\mathcal{O}(\lambda)$. Therefore, we expect a similar bound on M_{W_R} as derived in previous work [11]. However, the rapid progress in lattice QCD calculations warrants an update. When the QCD radiative corrections are taken into account explicitly, the above effective hamiltonian will be multiplied by an additional factor η_4 . [We neglect contributions of other operators with small coefficients.] In the leading-logarithmic approximation, η_4 is about 1.4 when the the four-quark operators are defined at the scale of 2 GeV in $\overline{\text{MS}}$ scheme [13].

The hadronic matrix element of the above operator can be calculated in lattice QCD and expressed in terms of a factorized form

$$\langle K_0 | \bar{d}(1 - \gamma_5)s \bar{d}(1 + \gamma_5)s | \bar{K}_0 \rangle$$

$$= 2M_K f_K^2 B_4(\mu) \left(\frac{m_K}{m_s(\mu) + m_d(\mu)} \right)^2. \quad (17)$$

Using the domain-wall fermion, one finds $B_4 = 0.81$ at $\mu = 2$ GeV in naive dimensional regularization (NDR) scheme [14]. In the same scheme and scale, the strange quark mass is $m_s = 98(6)$ MeV. Using the standard assumption that the new physics contribution shall be less than the experimental value, one finds

$$M_{W_R} > 2.5 \text{ TeV}, \quad (18)$$

which is now *the* bound in the model with the general CP violation. This bound is stronger than similar ones obtained before because of the new chiral-symmetric calculation of B_4 and the updated value of the strange quark mass.

The most interesting predictions of V_R are for CP-violating observables. We first study the CP violating parameter ϵ in K_L decay. When the SCPV phase $\alpha = 0$, the $W_L - W_R$ box diagram still makes a significant contribution to ϵ from the phase δ_{CKM} of the CKM matrix. The experimental data then requires W_R be at least 20 TeV to suppress this contribution. When $\alpha \neq 0$, it is possible to relax the constraint by cancelations. The most significant contribution due to α comes from the element V_{Rcd} which is naturally on the order of λ . In the presence of α , we have an approximate expression for ϵ_{LR}

$$\epsilon_{LR} = 0.77 \left(\frac{1 \text{ TeV}}{M_R} \right)^2 s_s s_d \text{Im} [g(M_R, \theta_2, \theta_3) e^{-i(\theta_1 + \theta_2)}] \quad (19)$$

where the function $g(M_R, \theta_2, \theta_3) = -2.22 + [0.076 + (0.030 + 0.013i) \cos 2(\theta_2 - \theta_3)] \ln \left(\frac{80 \text{ GeV}}{M_R} \right)^2$. The required value of $r \sin \alpha$ for cancelation depends sensitively on the sign of quark masses. When $s_s = s_d$, there exist small $r \sin \alpha$ solutions even when M_{W_R} is as low as 1 TeV, as shown by solid dots in Fig. 1. However, when $s_s = -s_d$, only large $r \sin \alpha$ solutions are possible.

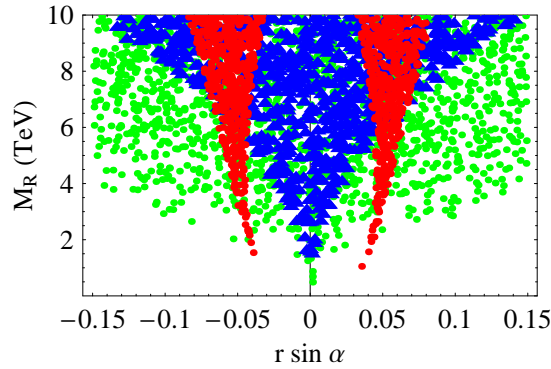


FIG. 1: Constraints on the mass of W_R and the spontaneous CP violating parameter $r \sin \alpha$ from ϵ (red dots) and neutron EDM in two different limit: small $r \sim 0.1$ (green dots) and large $r \sim 1$ (blue triangles).

An intriguing feature appears if one considers the constraint from the neutron EDM as well. A calculation of EDM is generally complicated because of strongly interacting quarks inside the neutron. As an estimate, one can work in the quark models by first calculating the EDM of the constituent quarks. In our model, there is a dominant contribution from the $W_L - W_R$ boson mixing [15]. Requiring the theoretical value be below the current experimental bound, the neutron EDM prefers small $r \sin \alpha$ solutions as can be seen in Fig. 1. The combined constraints from ϵ and the neutron EDM ($d_n < 3 \times 10^{-26}$ e cm [16]) impose an independent bound on M_{W_R}

$$M_{W_R} > (2 - 6) \text{ TeV} , \quad (20)$$

where the lowest bound is obtained for small $r \sim 0.05$ and large CP phase $\alpha = \pi/2$.

Finally we consider the neutral B -meson mixing and CP-violating decays. In $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing, due to the heavy b -quark mass, there is no chiral enhancement in the hadronic matrix elements of $\Delta B = 2$ operators from $W_L - W_R$ box diagram as in the

kaon case. One generally expects the constraint from neutral B -meson mass difference to be weak. In fact, we find a lower bound on W_R mass of 1-2 TeV from B -mixing. On the other hand, CP asymmetry in decay $B_d \rightarrow J/\psi K_S$, $S_{J/\psi K_S} = \sin 2\beta$ in the standard model, receives a new contribution from both $B_d - \bar{B}_d$ and $K_0 - \bar{K}_0$ mixing in the presence of W_R [7] and is very sensitive to the relative sign s_d and s_s . By demanding the modified $\sin 2\beta_{\text{eff}}$ within the experimental error bar, we find another independent bound on M_{W_R} ,

$$M_{W_R} > 2.4 \text{ TeV} , \quad (21)$$

when $s_d = s_s$ as required by the neutron EDM bound.

To summarize, we have derived analytically the right-handed quark mixing in the minimal left-right symmetric model with general CP violation. Using this and the kaon and B -meson mixing and the neutron EDM bound, we derive new bounds on the mass of right-handed gauge boson, consistently above 2.5 TeV. To relax this constraint, one can consider models with different Higgs structure and/or supersymmetrize the theory. A more detailed account of the present work, including direct CP observables, will be published elsewhere [17]

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